NOTATION

h_x, h_y, grid steps; ρ, density; u, v, velocity components; p, pressure; ε, internal energy; $E = \rho(\varepsilon + 0.5(u^2 + v^2))$, total energy; μ, viscosity coefficient; $c = \sqrt{\gamma(\gamma-1)\varepsilon}$ speed of sound; Re, Reynolds number; Pr, Prandtl number; γ, adiabatic index; T_w, all temperature; T_e, equilibrium wall temperature; M, Mach number; δ, thickness of the boundary layer; q, heat flux at the wall. Indices: ∞, parameters in the undisturbed oncoming stream; $\overset{0}{x}$, $\bar{x}$, x, ($\overset{0}{y}$, $\bar{y}$, y), central, left-hand, and right-hand difference derivatives with respect to x (and to y), respectively.

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DYNAMICS OF A BOUNDED GAS CAVITY IN A PIPE

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The dynamics of a gas cavity formed in a pipe in the zone of flow separation behind a cavitating body, and bounded by a diaphragm mounted at a pipeline exit, is considered. A mathematical model is proposed, on the basis of which the stability of the flow under consideration is investigated.

At present, more emphasis is being placed on the investigation of the dynamics of cavitation flows in pipes and flow elements of hydraulic systems. Of particular interest are flows with a developed connected cavity that is formed in the zones of flow separation behind the bluff elements of the structure or by special cavitating bodies. In these regions of flow with reduced pressure, a diffusive liberation of the dissolved gas or a buildup of dispersed gas bubbles that separate from the incoming flow takes place. When a certain critical cavitation number is realized, this results in the formation of a single cavity — a connected cavity with a sharply expressed gas—liquid interface. The adjustment of the dimensions of the middle sections of a cavitating body and a pipe, the blowing of the gas in the region of separation, as well as flow swirl, either natural or applied, affect substantially the intensity of these processes. When the cavitation number diminishes, the cavity dimensions increase along the flow, and flow lines at its interface tend to become parallel to each other and to the pipe walls in the limit [1-3].

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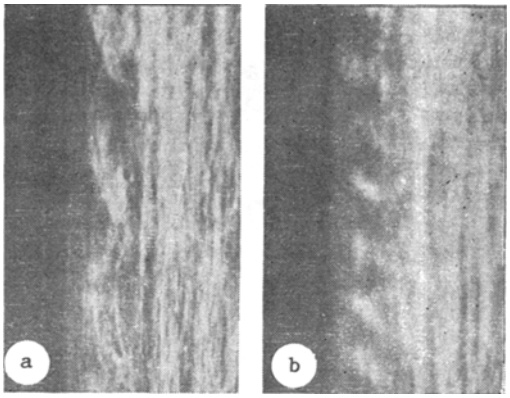

Fig. 1. Characteristic form of propagation of perturbations along the surface of the bounded gas cavity in the form of traveling waves for different values of vibration frequencies; (a) for f = 13.3 Hz; (b) for f = 76.6 Hz.

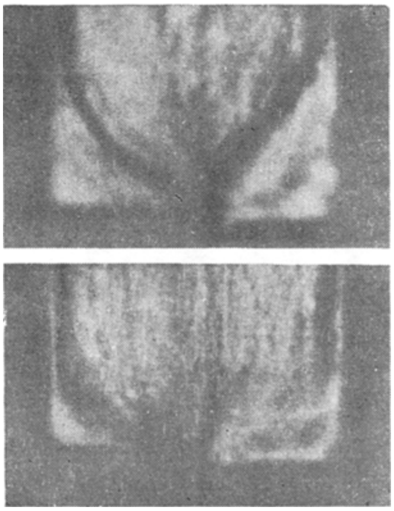

Fig. 2. Two extreme phases in the development of a tail piece of a cavity at the entrance of the flow in the local hydraulic resistance.

Experimental studies of these flows show that in many cases they are nonstationary. One of the external manifestations of a nonstationary nature is a periodic change in the axial and radial dimensions of cavity formations with respect to certain mean values. This can be both a result of a response of the cavity to external perturbations and a consequence of its inherent instability. It is necessary to note that the majority of existing investigations are devoted to analyses of dynamics of developed cavities formed in free or semi-bounded flows [1-5]. As distinguished from the cases under consideration for internal flows, real hydraulic systems are characterized by the presence of limits on the development of cavities, which arise due to the presence of turbulators, valves, diaphragms, grids, and other local hydraulic resistances located along the flow, in an axial direction.

The present work is devoted to the development of a mathematical model, describing the dynamics of the gas cavity that is formed in a pipe behind a bluff body bounded by a lumped hydraulic resistance of diaphragm type mounted at the exit of the pipe. On the basis of the proposed mathematical model we have the results of an analysis of visual observations and materials of high-speed filming, which have allowed us to exhibit characteristic features of the behavior of the cavity under consideration during vibrations. In particular, it has been found that the most significant changes in the volume occupied by the gas phase occur during fluctuations in the axial direction in the region immediately ahead of the entrance of the flow to the local hydraulic resistance. Radial deformations of the cavity in the form of waves traveling along the gas–liquid interface are less important. The characteristic pattern of propagation of perturbations on the surface of the cavity in the form of traveling waves is represented in Fig. 1. The photographs in Fig. 2 illustrate two extreme phases in the development of a tail piece of the cavity before the flow enters a local hydraulic resistance. It should be mentioned here that we do not consider regimes under which a progressively increasing development of the surface waves due to the instability of the gas–liquid interface might result in separation and ablation of the entire tail piece of the cavity, as is described, for example, in [1, 3], and also regimes with vortex cavity closure according to a mechanism with backwater until the flow enters a local hydraulic resistance [1, 4]. The diagram of the flow under consideration is shown in Fig. 3.

In the proposed model, we assumed that the intake of the gas phase in the cavity is provided by its forced supply in the zone of flow separation behind the cavitating body, via a pipe equipped with a distributed exit in order to avoid "the driving effect" [3]. A linearized equation for the variation in the gas mass in the cavity is of the form

$$\frac{d\delta M_g}{dt} = \delta G^g_{in} - \delta G^g_y, \tag{1}$$

where $\delta M_{gas} = \delta(\rho_g V_g)$ is the variation of the gas mass in the cavity.

A relationship between the gas density and pressure in the cavity under the assumption, for example, that the process is isothermal is determined from the equation

$$\frac{\delta P_g}{\delta \rho_g} = a_g^2, \tag{2}$$

where α_g is the speed of propagation of small perturbations in the gas.

In order to analyze the basic laws for the flow under consideration, we can represent a gas cavity by an equivalent cylinder with area F_c and length ℓ_c, the radial perturbations of the generatrix of which are described by the expression

$$\delta h = \delta h_0 \exp[jk(v_{ph}t - x)], \tag{3}$$

where $v_{ph} = v_0 \pm \sqrt{v_g^2 + v_\sigma^2}$ v_0 is the speed in the annular layer of the liquid surrounding the cavity. Here and below, the index 0 corresponds both to peak parameters and also to mean and constant parameters.

As distinguished from [6 and 7], where swirling flows were considered for which the phase velocity of gravitational waves due to the flow swirl can exceed substantially the axial velocity of the liquid layer, by limiting consideration to the low-frequency spectrum perturbations, we can assume that v_{gr} and $v_\sigma \ll v_0$. This condition represents, in particular, a criterion for the absence of waves moving along the region under consideration with the gas cavity up the flow, the emergence of which might be caused by the reflection of perturbations from the local hydraulic resistance that bounds the cavity. Therefore, the phase velocity of small perturbations in (3) is actually determined by the relationship $v_{ph} \approx v_0$. In correspondence with this, a variation in the flow rate in the layer surrounding the cavity can be written in the form

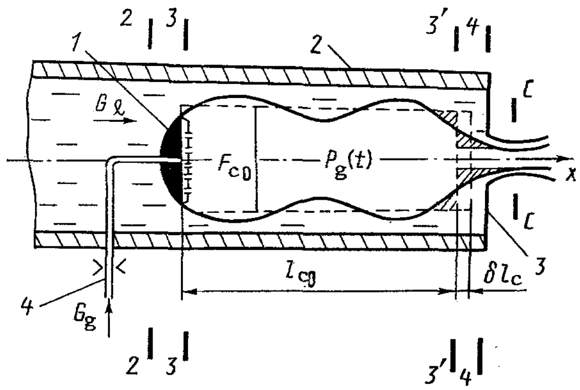

Fig. 3. Flow diagram with a bounded gas cavity: 1) cavitating body; 2) pipe walls; 3) diaphragm; 4) gas delivery pipeline.

$$\delta G^{\ell} = \delta G_0^{\ell} \exp[jk(v_0 t - x)]. \tag{4}$$

A linearized equation that described a variation in volume of the gas cavity in the axial direction was assumed to be as follows [4]:

$$\mu_c \frac{d^2 \delta V_g}{dt^2} = \delta P_g - \delta P_4, \tag{5}$$

where $\delta V_g = F_{c0} \delta \ell_c$; δP_4 is the variation in pressure in the cross-section 4-4 ahead of the entrance of the flow in the local hydraulic resistance (Fig. 3); $\delta P_g = \delta P_g(t)$ is the variation in pressure of the gas in the cavity, which was assumed to be the same throughout the entire volume [4, 5].

On the basis of Eq. (4), the relationship between the variations in the flow rate in cross sections 3-3 and 3'-3' of the pipeline (Fig. 3) is of the form

$$\delta G_{3'}^{\ell}(t) = \delta G_3^{\ell}(t - \tau), \tag{6}$$

where $\tau \approx \ell_{c0}/v_0$ is the delay time related to the propagation of a drifting perturbation wave in the area with laminar motion of phases.

For the liquid contained in the volume and bounded by the contours of the cavity and cross sections 3'—3' and C—C, we can write down

$$\frac{d\delta M_{\ell}}{dt} = \delta G_{3'}^{\ell} - \delta G_c^{\ell}, \tag{7}$$

where $\delta M_{\ell} = -\rho_{\ell} \delta V_g$; δG_3^{ℓ}, δG_C^{ℓ} are variations in flow rates through the cross section 3'—3' of the pipeline and the compressed cross section C—C of the discharging jet, respectively.

The gas from the cavity in the given case is carried out in the process of discharging of a two-phase mixture through the local hydraulic resistance mounted at the exit of a pipeline. By not going into the particulars of a complicated hydrodynamic pattern of this phenomenon (it will become the subject matter of an independent research), in order to obtain the results of the first approximation, we can use, for example, a model of discharge of a mixture with conditionally independent motion of phases for equal pressure differences along each of them. According to this model, the discharge of gas and liquid in the compressed cross section of the discharge jet depends on the area of the compressed cross section, occupied by each of the phases [8, 9]. In the given case, the following relation is applicable for the flow rate of the liquid:

$$\delta G_{C\text{-}C}^{\ell} = \mu F_0 (1 - \alpha) \sqrt{2 \Delta P \rho_{\ell}}, \tag{8}$$

here $\Delta P = P_4 - P_{C\text{-}C}$.

For the discharging gas phase for the case of a complete lamination of the flow in the compressed cross section, the flow rate can be determined from the equation

$$G_{out}^{g} = \mu F_0 \alpha \sqrt{\varkappa \left(\frac{2}{\varkappa + 1} \right)^{\frac{\varkappa+1}{\varkappa-1}} P_g \rho_g q(\lambda)}, \tag{9}$$

where $q(\lambda)$ is a gas dynamic function depending on the ratio $P_{C\text{-}C}/P_g$ [for the critical difference, $q(\lambda) = 1$].

Without taking account of the gas compressibility during the discharge, the equation for its flow rate assumes a simpler form

$$G_{out}^{g} = \mu F_0 \alpha \sqrt{2 \Delta P \rho_g}, \tag{10}$$

here $\Delta P = P_g - P_{C\text{-}C}$. As this takes place, α is determined from [9]:

$$\alpha = \left(\frac{G_C^{\ell}}{G_{out}} \sqrt{\frac{\rho_g}{\rho_{\ell}}} + 1 \right)^{-1}. \tag{11}$$

From (11), in particular, it follows that for $G_{C\text{-}C}^{\ell} \gg G_{out}^{g}$ $\alpha \ll 1$, i.e., the fraction of the cross section that is occupied by the discharge gas phase in this case is considerably less than the area occupied by the liquid. Then the variation in ∞ affects weakly the liquid flow rate, while the gas flow rate depends substantially on the variation in the flow area occupied by the gaseous phase. Therefore, when determining the variation in the phase discharge through the local hydraulic resistance the liquid can be conventionally considered as "a nonleaking valve" that closes the output cross section for the discharging gaseous phase, without an inverse effect of the variation in the area occupied by the gas phase in the compressed cross section, on the liquid flow rate [10]. In this case

$$\delta G^{r}_{out} = \gamma\delta P_g - N\delta G_C^{\ell}, \quad \delta G_C^{\ell} = K\delta P_4,$$
$$\gamma = \mu F_0 \sqrt{\frac{\rho_{g0}}{2\Delta P_0}}; \quad K = \mu F_0 \sqrt{\frac{\rho_\ell}{2\Delta P_0}}; \quad N = \sqrt{\frac{\rho_{g0}}{\rho_\ell}}. \tag{12}$$

The relationship among the parameters of the liquid flow at the input of the region with a bounded gas cavity is of the form

$$\delta P_2 - \delta P_g = \xi_c \rho_\ell v_{20} \delta v_2, \tag{13}$$

where $\delta v_2 = \delta G_2^{\ell}/\rho_\ell F_p$ is the variation in the liquid rate immediately ahead of the cavitating body in the cross section 2–2.

The system of equations (1), (2), (5)-(7), (12), and (13), completed by the equations of dynamics of the liquid in the pipeline and expressions for the variation in discharge of the gas, which is supplied into the cavity, is closed. In order to analyze the dynamics of the gas cavity inherently bounded in the pipeline, it is necessary to introduce certain assumptions that make the diagram of the process under consideration more specific. Thus, if we assume that the supply of gas in the cavity is provided by the blowing-in through the pipeline with a critical washer, mounted on it, i.e., $\delta G^g_{in} = 0$, and pressures ahead of the cavitator and at the exit beyond the diaphragm are constant, then from the above-mentioned system of equations we can obtain an equation that relates the variation in volume of a bounded gas cavity to the pressure of the gas contained in it:

$$A_3 \frac{d^3\delta V_g}{dt^3} + A_2 \frac{d^2\delta V_g}{dt^2} + A_1 \frac{d\delta V_g}{dt} = B\delta P_g(t-\tau), \tag{14}$$

where

$$A_3 = \mu_c \frac{V_{g0}}{a_g^2}; \quad A_2 = \gamma\mu_c + \frac{\rho_\ell V_{g0}}{K a_g^2};$$
$$A_1 = \rho_{g0}; \quad B = \frac{V_{g0}}{K a_g^2 \xi_c v_{20}}.$$

If, in order to obtain the results of the first approximation, as was done in [5], we assume that the gas mass in the cavity remains constant, i.e., $\delta M_g = 0$ then from the system of equations we can readily derive an equation describing the variation in volume of the cavity in the form

$$a_2 \frac{d^2\delta V_g}{dt^2} + a_1 \frac{d\delta V_g}{dt} + a_0\delta V_g + e_0\delta V(t-\tau) = 0, \tag{15}$$

where

$$a_2 = \mu_c; \quad a_1 = \frac{\rho_\ell F_{c0}}{K};$$
$$a_0 = \frac{\rho_{g0} a_g^2}{l_{c0}}, \quad e_0 = \frac{F_p \rho_{g0} a_g^2}{K \xi_c v_{20} l_{c0}}.$$

A characteristic feature of the solution obtained is the presence in it of the term with a deviating argument. This allows us to assume that there are nonsteady regimes in the cavity flow under consideration.

The characteristic equation corresponding to (15) is of the form [11]

$$r^2 + \varepsilon r + \lambda_0 + \lambda_1 \exp(-r\tau) = 0, \tag{16}$$

where $\varepsilon = a_1/a_2$; $\lambda_0 = a_0/a_2$; $\lambda_1 = e_0/a_2$.

Equation (16) allows us, with the help of the method of D-division, to determine the boundaries of the regions of stability in the plane of any two parameters. If we let $r = j\omega$ in (16), we obtain, for example, parametric equations for the domain boundaries of the D-division over the plane (τ, λ_1). In this case, if we set equal to zero the (Re S) and imaginary (Im S) parts of the characteristic equation (16), we obtain

$$-\omega^2 + \lambda_0 + \lambda_1 \cos(\omega\tau) = 0,$$
$$\varepsilon\omega - \lambda_1 \sin(\omega\tau) = 0, \tag{17}$$

or, alternately,

$$\tau = \frac{1}{\omega} \operatorname{arctg} \frac{\varepsilon\omega}{\omega^2 - \lambda_0} + \frac{n\pi}{\omega},$$
$$\lambda_1 = (\omega^2 - \lambda_0) / \cos(\omega\tau), \tag{18}$$
$$n = 0, \pm 1, \pm 2, \ldots$$

The location of the domains of stability and instability over the plane (τ, λ_1) with respect to boundaries of the D-division is related to the sign of the determinant

$$\Delta = \begin{vmatrix} \frac{\partial \operatorname{Re} S}{\partial \tau} & \frac{\partial \operatorname{Re} S}{\partial \lambda_1} \\ \frac{\partial \operatorname{Im} S}{\partial \tau} & \frac{\partial \operatorname{Im} S}{\partial \lambda_1} \end{vmatrix} = \lambda_1 \omega.$$

If $\Delta > 0$, while ω varies from $-\infty$ to $+\infty$, then the D-division boundary is hatched from the left; for $\Delta < 0$, in the reverse direction.

A typical pattern for the boundaries of domains of the D-division, corresponding to calculation for $\lambda_0 = 10^4$ sec^{-2} in the range of physically realized values of parameters is represented in Fig. 4. When τ and λ_1 increase, domains of the D-division become multiply connected; in Fig. 4 they are denoted by D_2, D_3,..., which attests that the system can lose the stability at several frequencies simultaneously [11].

In order to obtain a more illustrative representation of the vibrations of a bounded gaseous cavity in a pipe for the case when it is unstable, we expand the term $\delta V_g(t - \tau)$, assuming that τ is a small parameter, in a power series

$$\delta V_g(t - \tau) = \delta V_g(t) - \frac{d\delta V_g}{dt} \frac{\tau}{1!} + \frac{d^2 \delta V_g}{dt^2} \frac{\tau^2}{2!} - \ldots \tag{19}$$

If we retain only the two first terms in (19), then from (15) we can obtain

$$\frac{d^2 \delta V_g}{dt^2} + 2b \frac{d\delta V_g}{dt} + \omega_0^2 \delta V_g = 0, \tag{20}$$

where $2b = \varepsilon - \lambda_1 \tau$; $\omega_0^2 = \lambda_0 + \lambda_1$.

Therefore, since $b < 0$ in the region of instability the emergence of an instability is possible; such an instability exhibits itself in the form of periodic fluctuations of the gas cavity that increase exponentially. As is seen from (20), an instability is possible when $\varepsilon < \lambda_1 \tau$, i.e., in fact, it is determined by the length of the path on which the bounded

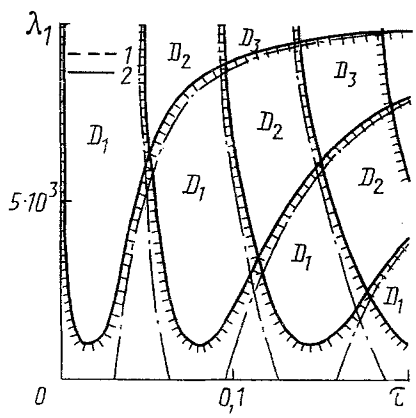

Fig. 4. Boundaries of the domains of D-division in the plane of parameters λ_1, sec^{-2}; τ, s (dot-dash curves, calculation for $\varepsilon = 0$; continuous curves, calculation for $\varepsilon = 10$ sec^{-1}).

gas cavity exists, and the phase velocity of motion of the perturbation wave. Physically, the emergence of the instability is related to the delay time of the pressure variation in the gas cavity with respect to the variation in the flow rate of the liquid that is delivered to the cavitating body's entrance and determines this variation by closing the flow cross section for the discharging gas phase.

Conclusions. A mathematical model has been developed that reflects the characteristic features of the dynamics of a bounded gas cavity in a pipeline, which are obtained from visual and photographic investigations, and discloses the physical nature of the emergence of self-exciting vibrational processes in the system. The regions of instability in the plane of operating conditions of the flow are found.

NOTATION

M, V. P, ρ, v, and G, mass, volume, pressure, density, velocity, and mass flow rate of the phases, respectively; $j = \sqrt{-1}$; k, wave number; α_g, sound velocity in the gas; v_{ph}, phase velocity of the perturbation wave; v_σ, v_{gr}, phase velocities of purely capillary and gravitational waves at the phase interface; x, longitudinal coordinate; t, time; τ, delay time; μ_c, coefficient of connected mass of a cavity; μ, coefficient of contraction of a discharging jet; ξ_c; resistance coefficient of a cavitating body; F_0, area of the flow cross section of a diaphragm; F_p, area of the middle section of a pipe; χ, adiabatic index. Indices: g, gas phase; ℓ, liquid; 2, 3, 3', and 4 refer to the corresponding cross sections 2–2, 3–3,... of the flow; C corresponds to the parameters in the contracted cross section C–C of the discharging jet; 0 corresponds to the peak, mean, and fixed parameters; a, carrying away or removal of a phase; δ, symbol of variation of the corresponding parameter.

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